

Math 53 Discussion Problems Nov 12

1. Find the Jacobian for the following transformations.

(a) $x = u + 2v, y = u - v$

(b) $x = u^2 - v^2, y = 2uv$

(c) $x = r \cos \theta, y = r \sin \theta$

(d) $x = \rho \sin \phi \cos \theta, y = \rho \sin \phi \sin \theta, z = \rho \cos \phi$

2. Evaluate the integral $\iint_R (2x^2 - xy - y^2) dx dy$ where R is the region in the first quadrant bounded by the lines $y = -2x + 4, y = -2x + 7, y = x - 2, y = x + 1$, using the transformation $u = x - y, v = 2x + y$.

3. Evaluate the integral $\iint_R (\sqrt{\frac{y}{x}} + \sqrt{xy}) dx dy$ where R is the region in the first quadrant bounded by the hyperbolas $xy = 1, xy = 9$ and the lines $y = x, y = 4x$, using the transformation $x = \frac{u}{v}, y = uv$ with $u > 0, v > 0$.

4. Evaluate the integral $\iiint_D (x^2y + 3xyz) dx dy dz$ where D is the region defined by the inequalities $1 \leq x \leq 2, 0 \leq xy \leq 2, 0 \leq z \leq 1$, using the transformation $u = x, v = xy, w = 3z$.